Chapter 16 Small Sample Hypothesis Testing Using Student's t Test

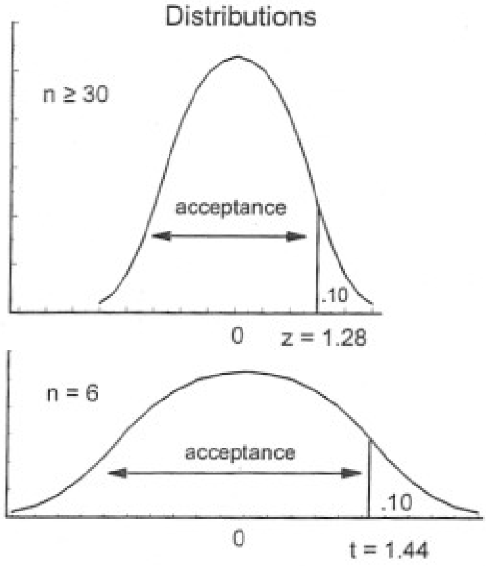
I. **Large versus small samples**

A. The standard normal distribution (z) is appropriate for large samples ($n \geq 30$). The population may be normal or skewed.

1. If σ is unknown, use s.
2. For small samples, $n < 30$, z is appropriate provided the population is normal and σ is known.

B. The student t distribution is appropriate for small samples, $n < 30$, provided the population is normal, and σ is not known (use s).

C. Small skewed distributions will be discussed in chapter 20.



Comparing z and t Distributions

II. **The t distribution's characteristics**

A. The t distribution is a family of distributions.

1. A distribution's **degrees of freedom (df)** is determined by the number of samples involved with the distribution and the size of these samples.
2. Degrees of freedom and level of significance determine t values.

B. The t distribution is approximately normal and flatter than the z distribution.

C. Values for t are larger than their corresponding z values, though the difference is negligible when n is over 29. Some statistics software use t values even when n is larger than 29.

III. **One-tail testing of one sample mean using t**

A. Linda wants to know whether average tapes rented per customer has decreased from last year's mean of 2.6 tapes. A recent sample of 9 customers had a mean of 2.3 tapes and a standard deviation of .3. Test at the .01 level of significance whether average tape rentals decreased. Assume a normal distribution.

B. The 5-step approach to hypothesis testing

1. Here are the null hypothesis and alternate hypothesis.

$$H_0 : \mu \geq 2.6 \text{ and } H_1 : \mu < 2.6$$

2. The level of significance is .01.
3. The relevant statistic is $\bar{x}$.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

4. Reject the null hypothesis when t from the test statistic is beyond t's critical value.

   a. When testing one mean, there are n - 1 degrees of freedom.

$$n - 1 = 9 - 1 = 8$$

   b. The critical value of t is -2.896 for the .01 level.

5. Apply the decision rule.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{2.3 - 2.6}{\frac{.3}{\sqrt{9}}} = \frac{-.3}{\frac{.3}{3}} = -3.0$$

Reject H_0 because -3.0 is beyond -2.896.
Average tape rentals decreased.

Partial Student t Distribution

Degrees of freedom df	Area from the mean to a critical value				
	0.40	0.45	0.475	0.49	0.495
	α for a one-tail problem				
	0.10	0.05	0.025	0.01	0.005
	α for a two-tail problem				
	0.20	0.10	0.05	0.02	0.01
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.883	2.262	2.821	3.250
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750

See page ST 4 for a more complete t table.

IV. **Two-tail testing of one sample mean using t**

A. This test involves measuring change in either direction.

B. Procedures are the same as those described in earlier chapters.