Chapter 22 Nonparametric Hypothesis Testing of Ordinal Data Part II

I. **Two-tail testing of 2 sample medians from dependent populations using a paired difference sign test**

A. This test is equivalent to a two-tail parametric test for statistical dependence. (see part VI page 99)

B. Data must be at least ordinal in nature. Knowledge of the sampling distribution's shape is not necessary.

C. The test uses a (+) sign to represent situations where the first variable is larger than the second variable. It uses a (-) sign to represent the opposite situation. Zero represents a situation where variables are equal. Zero values are excluded from the test. Each time this happens, the sample size is reduced by one.

D. If the medians are equal, the proportion of (+) signs should be approximately equal to the proportion of (-) signs.

1. H_o: p = .50 and H_1 : p ≠ .50
2. For small samples, we use the binomial distribution to determine the likelihood of one of the signs occurring a large number of times. P is equal to .5 and n is equal to the number of observations. If the probability, based upon the observed signs, is greater than the level of significance, the null hypothesis is accepted. Z may be used for large samples with p = .50. (see IC of page 94)

E. Weekly sales before and after a big promotion at three of Linda's stores were $1,200, $1,300 and $1,400 and $1,400, $1,500 and $1,500 respectively. This data was first studied on page 99. At that time, it was assumed the populations were normal. If this were not the case or unknown, a .10 level sign test of the median could have been conducted.

Store	Sales Dollars Before	Sales Dollars After	Sign
1	1,200	1,400	+
2	1,300	1,500	+
3	1,400	1,500	+

F. The p-value approach to hypothesis testing is used for these sign tests.

1. This table indicates median sales increased at all 3 stores. Sample size is 3.
2. The Binomial table (ST 1) yields the following: P(x ≥3) = .125.
3. For this two-tail test, p = (.125)(2) = .250. Accept the null hypothesis because .25>.10. Medians are equal.
4. This null hypothesis can't be rejected because the sample size is too small.

G. One- and two-tail brand preference tests can be done with a paired difference sign test.

II. **Testing 3 or more sample medians from independent populations using the Kruskal-Wallis test**

A. The ANOVA analysis of chapter 18 required populations be normally distributed with equal variances. If these requirements are not met or unknown, the parametric ANOVA test of several means is replaced with the nonparametric Kruskal-Wallis H test of several medians.

B. This test complements the Mann-Whitney test of 2 medians.

C. This test requires that data from independent random samples be at least ordinal in nature.

D. Data is ranked. Ties are assigned the average of their ranks. A true null hypothesis means average group ranks are approximately equal. Special tables, not provided here, should be used if n < 5.

E. The chapter 18 salesperson's sales data, with a week added so n = 5, will be tested for equality of medians at the .05 level of significance. We will not assume normal distributions and use the Kruskal-Wallis test.

F. Weekly sales data is ranked with this chart.

$$H = \frac{12}{N(N+1)}\left[\frac{(\sum R_1)^2}{n_1} + \frac{(\sum R_2)^2}{n_2} + \dots + \frac{(\sum R_k)^2}{n_k}\right] - 3\,(N+1)$$

Where: H is the designated statistic.
k is the number of samples.
N is the number of observations.
n_k is a sample's size.
R_k is a sample's rank total.
df = k - 1 = 3 - 1 = 2 → $\chi^2 = 5.99$

Weekly Sales (x) in Thousands of Dollars

Salesperson L		Salesperson M		Salesperson N	
Sales	Rank R_1	Sales	Rank R_2	Sales	Rank R_3
7	9.5	6.0	5.5	9.0	14.0
6	5.5	8.0	12.5	8.0	12.5
7	9.5	6.0	5.5	7.0	9.5
4	2.0	6.0	5.5	10.0	15.0
3	1.0	5.0	3.0	7.0	9.5
	R_1= 27.5		R_2= 32.0		R_3= 60.5

$$H = \frac{12}{15(15+1)}\left[\frac{(27.5)^2}{5} + \frac{(32)^2}{5} + \frac{(60.5)^2}{5}\right] - 3\,(N+1)$$

$$= .05[151.25 + 204.80 + 732.05] - 3(15+1)$$

$$= 54.405 - 48.000 = 6.405$$

Reject H_0 because H_0 = 6.41 > 5.99. Medians are not equal.

Notes: 1) An adjustment, not shown here, is required when there are many ties.
2) Both the Mann-Whitney test and the Kruskal-Wallis test require populations be of similar shape and dispersion.