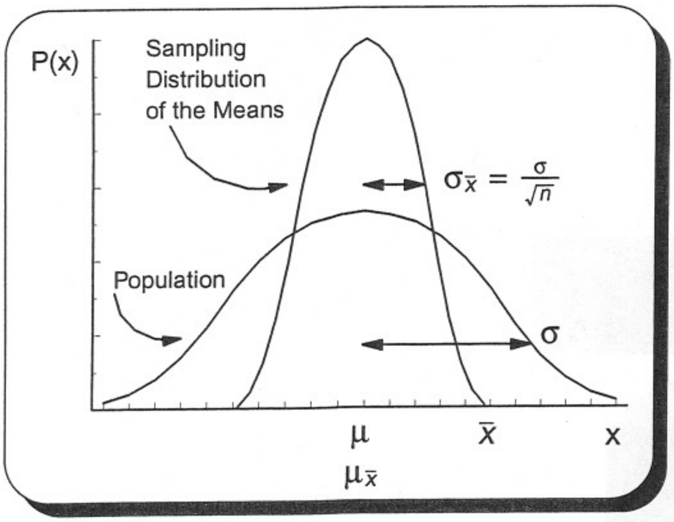
V. Central limit theorem

A. **The sampling distribution of the means will be normal whenever the population is normal.**

B. The central limit theorem also applies to skewed populations provided the sample is large ($n \geq 30$).

C. The relationship between the parameters of a population and its sampling distribution is shown below.



Note: Because the sampling distribution is normal regardless of its population's skewness, a sampling distribution's mean can be used to make predictions about one of its sample means. Prediction procedures will be similar to those followed in chapter 10, where the population mean was used to make predictions about a value of x. In practice, the sample mean is known and used to make estimates about the sampling distribution's mean. These estimates also apply to the population mean because said means are equal. These estimates of a population mean can be very accurate because the sampling distribution's standard deviation will be smaller if the sample size is increased. Diminishing returns apply to larger samples being more accurate as the denominator of $\sigma_{\bar{x}}$ is not n but the square root of n. A sample of 49 is only slightly more accurate than a sample of 36. Why? Because the denominator is only slightly larger (7 vs. 6), and the sampling distribution's standard deviation is not proportionately smaller.

VI. Using a large sample ($n \geq 30$) to determine point and interval estimates of population parameters

A. **Point estimates**
1. A point estimate is a one-number estimate.
2. Important point estimates
   a. A sample mean for its population mean
   b. A sample standard deviation for its population standard deviation

Section B Note: When $n < 30$ and σ is unknown, the t distribution, discussed in chapter 16, must be substituted for the z distribution when making interval estimates. Many statistics software programs do all interval calculations, regardless of sample size, using the t distribution.

B. **Interval estimates**
1. An interval estimate is a range.
2. A range for μ, called a **confidence interval**, is determined using this expression. $\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
3. The standard deviation of a sampling distribution ($\sigma_{\bar{x}}$), called the **standard error of the mean**, is very important in determining an interval estimate of a population mean.
4. Below are two important confidence intervals for $\mu_{\bar{x}}$ and therefore μ.
   a. 95 percent confidence interval

      z for .95/2 = .4750 → 1.96 $\quad \bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

   b. 99 percent confidence interval

      z for .99/2 = .4950 → 2.58 $\quad \bar{x} \pm 2.58\frac{\sigma}{\sqrt{n}}$

Note: These interval estimates are based upon the relationship between z, the population distribution, and the sampling distribution of the means.

$$z = \frac{x-\mu}{\sigma} = \frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$$

5. When population standard deviation is unknown, the sample standard deviation may be used as a point estimate of the population standard deviation provided the sample is large. Small samples will be examined in chapter 16.

$$\bar{x} \pm 2.58\frac{s}{\sqrt{n}}$$

C. **Example:** Linda took a random sample of 49 customer orders and found the mean purchase amounted to \$7.50. The population standard deviation is known to be \$.70. The 99% confidence interval for the population mean purchase has been calculated in this frame.

Note: Linda can lower the range by accepting a confidence interval of only 95% or by increasing the sample size.

Given:
$\bar{x}$ = \$7.50
σ = \$.70
n = 49
z for .99 is 2.58

$\bar{x} \pm 2.58\frac{\sigma}{\sqrt{n}}$

$\$7.50 \pm 2.58\frac{\$.70}{\sqrt{49}}$

$\$7.50 \pm 2.58(\$.10)$

$\$7.50 \pm \$.258$

$7.24 ↔ $7.76