III. Determining sample size

A. A small sample may give an inadequate answer (too large a confidence interval).

B. A large sample requires excess time and money.

C. Three factors are used to determine an appropriate sample size.

1. The population variance (σ^2)
   a. A large population variance means a larger sample is needed to yield acceptable results.
   b. If the population variance is not known, it may be estimated with a small preliminary survey.
2. The required degree of confidence (z)
   a. A given confidence interval (90%) has a matching **degree of confidence.** In the long run, there is a 90% degree of confidence that the population parameter being measured will fall within the 90% confidence interval.
   b. A higher degree of confidence requires a larger sample.
3. The amount of acceptable error (E)
   a. A study will have some logical acceptable range for the confidence interval.
      1) Income may be estimated to within $500 of the mean.
      2) A part's size may be estimated to within .01 millimeters.
   b. A small acceptable error requires a larger sample.

D. Sample size determination when estimating the population mean

1. Solving $\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$ for n gives the following sample size formula.
2. $$n = \left(\frac{z\sigma}{E}\right)^2$$
   Note: A large degree of confidence, a large variance, and a small acceptable error all make the sample size larger.
3. Suppose Linda was unhappy with the average customer purchase range first described on page 67 and summarized below. How large a sample would be required to lower the acceptable error from $.26 to $.10? Assume the finite correction factor is not applicable.

Problem Review

Given:
$\bar{x}$ = $7.50
z for .99 is 2.58
σ = $.70
n = 49

$$\bar{x} \pm 2.58\frac{\sigma}{\sqrt{n}}$$
$$\$7.50 \pm 2.58\frac{\$.70}{\sqrt{49}}$$
$$\$7.50 \pm \$.258$$
$$\$7.24 \leftrightarrow \$7.76$$

$$n = \left(\frac{z\sigma}{E}\right)^2$$
$$= \left[\frac{(2.58)(.7)}{.1}\right]^2$$
$$= [18.06]^2 = 326.16 \rightarrow 327$$
Note: always round up

4. Check your answer by calculating the confidence interval using the new sample size. If the interval is acceptable (within $.10), conduct your new survey with a sample of 327.
5. When determining the sample size for both mean and proportion problems, answers less than 30 should be rounded up to 30 because sample size formulas are based upon a normal population.

$$\bar{x} \pm Z\frac{\sigma}{\sqrt{n}}$$
$$\bar{x} \pm 2.58\frac{\$.70}{\sqrt{327}}$$
$$\bar{x} \pm .09987$$
and $.09987 < .10$

E. Sample size determination when estimating the population proportion

1. $$n = \bar{p}(1-\bar{p})\left(\frac{Z}{E}\right)^2$$
2. Using the problem II data from the previous page, Linda would like to lower the acceptable error associated with the 95% confidence interval for customer satisfaction from ± 7.45% to ± 5%. What sample size is required?
3. The sample size formula must include the page 70 finite correction factor because n/N is > .05.
4. From these calculations, it appears that Linda can reduce the range of the confidence interval to ± 5% by increasing the sample size to 234.
5. If $\bar{p}$ is not known, it may be estimated with a sample of 100. Also, using $\bar{p}$ of .5 will give the maximum appropriate sample size.

$$n = \bar{p}(1-\bar{p})\left(\frac{Z}{E}\right)^2\sqrt{\frac{N-n}{N-1}}$$
$$= .80(1-.80)\left(\frac{1.96}{.05}\right)^2(.949)$$
$$= .80(.20)(39.2)^2(.949)$$
$$= 233.3 \rightarrow 234$$