Chapter 12 Sampling Distributions Part II

I. Estimating the population proportion

A. The population proportion is the average part of a population having a particular trait.
B. It may be expressed as a fraction, decimal, or percentage.
C. The sample proportion is $\bar{p} = \frac{x}{n}$.
D. The population proportion is used to measure traits such as consumer attitudes toward a product, voter preference, and the proportion of parts passing inspection.
E. Experiments described here must meet the binomial experiment conditions described on page 52 and the normal approximation of the binomial conditions described on page 61.
F. Estimating a confidence interval for the population proportion using a large sample is explained below.

$$p = \frac{\text{successes}}{\text{population size}}$$

Some texts use π for the population proportion.

1. $\bar{p} \pm z\sigma_{\bar{p}}$ where $\sigma_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ and
   1) $\bar{p}$ is the sample proportion
   2) n, the sample size, is ≥ 30
   3) z is based upon the desired confidence interval

2. Example: Linda Smith randomly called 100 customers and found that 80 were happy with the service they received when shopping at Linda's Video Showcase. Calculate a 95% confidence interval for the population proportion. Given: $n = 100$ and z for 95% confidence is 1.96

$$\bar{p} = \frac{x}{n} = \frac{80}{100} = .80$$

$n = 100 \geq 30$

$np = 100 \times .8 = 80 \geq 5$

$nq = 100 \times .2 = 20 \geq 5$

The normal approximation of the binomial applies.

$$\sigma_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.8(1-.8)}{100}} = \sqrt{.0016} = .04$$

$$\bar{p} \pm z\sigma_{\bar{p}}$$

$.80 \pm 1.96(.04)$

$.80 \pm .0784$

$.722 \leftrightarrow .878$

II. Finite correction factor

A. Thus far, formulas used to calculate the **standard error of the mean** ($\sigma_{\bar{x}}$) and the **standard error of the proportion** ($\sigma_{\bar{p}}$) have been based upon infinitely large populations.
B. If the population is finite, then the relative size of our sample has increased, and the standard error can be reduced using the finite correction factor.

$$\sqrt{\frac{N-n}{N-1}}$$

C. The finite correction factor is used to calculate the standard error when $\frac{n}{N} \geq .05$. Smaller ratios are immaterial.

Standard Error of the Mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

Standard Error of the Proportion

$$\sigma_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \sqrt{\frac{N-n}{N-1}}$$

D. Linda must adjust her interval calculation because her customer pool totaled 1,000.

$$\frac{n}{N} = \frac{100}{1,000} = .10 \geq .05$$

$$\sigma_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \sqrt{\frac{N-n}{N-1}}$$

$$= .04\sqrt{\frac{1,000-100}{1,000-1}}$$

$$= .04\sqrt{.9009}$$

$$= .038$$

$$\bar{p} \pm z\sigma_{\bar{p}}$$

$.80 \pm 1.96(.038)$

$.80 \pm .0745$

$.726 \leftrightarrow .875$

Note: Because the range is slightly smaller, the prediction may be more useful.