III. A 5-step approach to hypothesis testing

A. State the null hypothesis and alternate hypothesis.
   1. Determine the condition (claim, concern, difference) being tested using >, <, or ≠. Call it H_1.
   2. Determine the condition's complement using ≤, ≥, or =. Call it H_0.
   3. H_0 implies no difference by containing an equality sign. It is stated first.

B. Select the level of significance based upon acceptable type I error.

C. Determine the relevant test statistics ($\bar{x}$ for now, $\bar{p}$ and others will follow).

D. Determine the decision rule using a graph of the critical values of z.
   1. Accept the null hypothesis if the test statistic z is not beyond the critical value of z.
   2. Otherwise, reject the null hypothesis.

E. Apply the decision rule.

Simply put, if the test statistic is extreme enough, beyond the critical value, reject the null hypothesis.

IV. One-tail testing of one sample mean

Linda Smith thinks average customer purchases could be lower than last year's \$7.75 because a sample of 49 (see page 67) had a mean of only \$7.50. The population standard deviation is \$.70. Linda wants type I error, the chance of rejecting a true null hypothesis, to be 1%.

A. $H_0 : \mu \geq \$7.75$ and $H_1 : \mu < \$7.75$

B. Type I error is 1%.

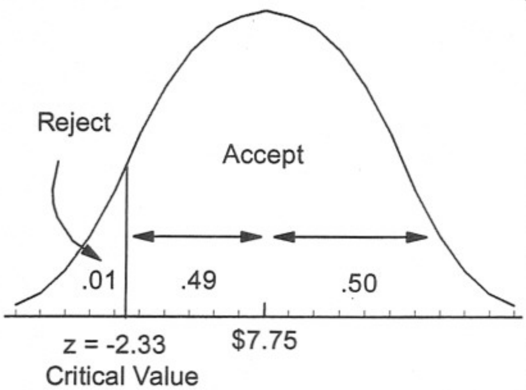
C. The test statistic is $\bar{x}$.

D. If z from the test statistic is beyond -2.33, reject the null hypothesis.

E.

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\$7.50 - \$7.75}{\frac{\$.70}{\sqrt{49}}} = \frac{-\$.25}{\frac{\$.70}{7}} = \frac{-.25}{.10} = -2.50$$

Reject the null hypothesis because a z of -2.50 is beyond (smaller) the critical value of -2.33. A sample mean of \$7.50 happens less-than 1% of the time when $\mu \geq \$7.75$.



Note: If the area beyond the test statistic (the tail) is less-than the level of significance, the measured difference is significant and H_0 is also rejected. This approach, called p-value hypothesis testing, is used by most statistics software. After completing this page, statistics software users should read part II of page 88 and chapter 16.

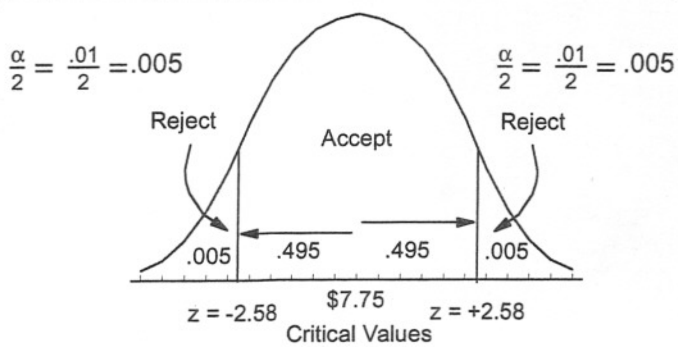
V. Two-tail testing of one sample mean

A. **Two-tail problems** concern any change, regardless of direction.

B. In the problem above, Linda was not concerned about the average purchase going up. Now she is. The claim concerning the average purchase must be changed to include any difference from last year's average purchase of \$7.75. The null hypothesis and alternate hypothesis would be: $H_0 : \mu = \$7.75 \qquad H_1 : \mu \neq \$7.75$

C. For this two-tail problem, the alternate hypothesis does not state the direction of the change (difference).

D. Using a .01 level of significance, alpha risk must be divided evenly between the 2 tails of a normal curve.



The test statistic remains ± 2.50. The analysis to the left indicates the critical value has changed to ± 2.58. **Accept** the null hypothesis as z of -2.50 is not beyond the critical value of -2.58. At the .01 level of significance, a sample mean of \$7.50 is not low enough to conclude the population mean is not \$7.75. Note how splitting the .01 level of significance (risk) between two tails increases the critical value. As a result, what was a significant difference is now an acceptable difference.