Practice Set 15 Hypothesis Testing of Population Proportions

I. Page 72 data showed that 90% (45 of 50) of the 30-milligram parts, taken from a lot of 1,000 parts, passed inspection. Darin wants a .01 level of significance test to determine whether the population proportion of parts passing inspection has increased from the 86% reported last year.

Given: n equals 50, $p = .86$, and $\bar{p} = .90$

The normal approximation to the binomial applies.

$$n = 50 > 30$$
$$np = 50(.86) = 43 \geq 5$$
$$nq = 50(1 - .86) = 7 \geq 5$$

1. The null hypothesis and alternate hypothesis are $H_0 : p \leq .86$ and $H_1 : p > .86.$
2. The level of significance will be .01.
3. The test statistic is $\bar{p}$.
4. If z from the test statistic is beyond the critical value of z, the null hypothesis will be rejected.
5. Apply the decision rule.

$$z = \frac{\bar{p}-p}{\sigma_p} = \frac{\bar{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.90-.86}{\sqrt{\frac{.86(1-.86)}{50}}} = \frac{.04}{.0491} = .81$$

Accept H_0 because .81 is not beyond 2.33. The proportion of parts passing inspection is not higher than last year.

II. Darin wants to determine at the .01 level of significance whether there is a difference in the proportion of defects produced during the day and night shifts. A sample of 100 parts was taken from each shift. The day shift had 5 defects and the night shift had 14 defects. Is there a difference in the proportion of defects produced by these two shifts?

Given:	$n_1 = 100$	$n_2 = 100$	$x_1 = 5$	$x_2 = 14$	$\alpha = .01$ and $.01/2 = .005 \rightarrow z = \pm 2.58$

$$p_1 = \frac{x_1}{n_1} = \frac{5}{100} = .05$$

$$p_2 = \frac{x_2}{n_2} = \frac{14}{100} = .14$$

$$\bar{p}_w = \frac{x_1 + x_2}{n_1 + n_2} = \frac{5+14}{100+100} = .095$$

The 5-step approach to hypothesis testing

1. The null hypothesis and alternate hypothesis are: H_0: $p_1 = p_2$ and H_1: $p_1 \neq p_2$
2. The level of significance will be .01.
3. The test statistic will be $\bar{p}$.
4. If z from the test statistic is beyond the critical value of z, the null hypothesis will be rejected.
5. Apply the decision rule.

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\frac{\bar{p}_w(1-\bar{p}_w)}{n_1} + \frac{\bar{p}_w(1-\bar{p}_w)}{n_2}}} = \frac{.05-.14}{\sqrt{\frac{.095(1-.095)}{100} + \frac{.095(1-.095)}{100}}} = \frac{-.09}{.0415} = -2.17$$

Accept H_0 because -2.17 is not beyond -2.58.
The defects proportion is the same for these two shifts.