Chapter 19 Two-Factor Analysis of Variance

I. **Sources of variability**
   A. Variance between treatment variables exists because treatments are not alike.
   B. Variance within a treatment is unexplained and due to sampling error.
   C. Additional sources of variability (called factors or treatments) may be added to a study.
      1. Their variability may be used to reduce unaccounted for, within treatment variability (error).
      2. Additional treatments are called **blocking variables**.
      3. They represent a substantial source of inherent response variability.
      4. Treatments must not be independent. Treatment B may affect the factors of treatment A differently. For example, weeks of experience may have a different affect on each of the recently hired sales people.
      5. Examples of blocking variables include age, gender, education, and time.

II. **Two-factor variance analysis**
   A. In chapter 18, Linda found that her 3 salespeople had different mean weekly sales and that half of the data's variability could be attributed to the salespeople treatment.
   B. Chapter 18 sales data was randomly assigned to each salesperson. Here, it has been arranged by weeks of experience. Using experience as a blocking variable may account for some of the unexplained variability. Treatments are not independent because weeks of experience may affect salespeople differently.
   C. $\sum x_B$ is the sales associated with each block (week). Number of treatments is now t, b is the number of blocks.

Weekly Sales (x) in Thousands of Dollars							Row Totals Required for Calculations		
Block(B_x)	Salesperson L is T_1		Salesperson M is T_2		Salesperson N is T_3				
Weeks	Sales(X_1)	X_1^2	Sales(X_2)	X_2^2	Sales(X_3)	X_3^2	$\sum x_B$	$(\sum x_B)^2$	$\frac{(\sum x_B)^2}{t}$
1	4	16	6	36	7	49	17	289	96.3
2	6	36	6	36	8	64	20	400	133.3
3	7	49	6	36	9	81	22	484	161.3
4	7	49	8	64	10	100	25	625	208.3
							$84 = \sum x$	$\sum[\frac{(\sum x_B)^2}{t}] =$	599.3
							$84 = \sum x$		
$\sum x_T$	24		26		34				
$(\sum x_T)^2$	576		676		1156				
b	4		4		4		N = 12		
$\frac{(\sum x_T)^2}{b}$	144		169		289		$\sum[\frac{(\sum x_T)^2}{b}] = 602$		
$\sum x_T^2$		150		172		294	$\sum x^2 = 616$		

Variance Analysis Summary Table				
Variance Sources	df	Sum of the Squares	Mean Squares	ANOVA
Between Treatments	t - 1	SS_T	$MS_T = \frac{SS_T}{t-1}$	$F = \frac{MS_T}{MS_E}$
Block	b - 1	SS_B	$MS_B = \frac{SS_B}{b-1}$	$F = \frac{MSB}{MS_E}$
Within Treatments (error)	(t - 1)(b - 1)	SS_E	$MS_E = \frac{SS_E}{(t-1)(b-1)}$	
Total Variance	N - 1	SS_{TOTAL}		

Note: This analysis is called "mean square" because it is based upon the variance.