IV. Linda Smith is using ANOVA to measure whether there is a difference between the average weekly sales of her 3 salespeople. The test will be at the .05 level of significance.

A. These are the null hypothesis and alternate hypothesis.

\[ H_0: \mu_1 = \mu_2 = \mu_3 \quad H_1: \mu_1 \neq \mu_2 \neq \mu_3 \]

B. The level of significance for this single treatment, one-tail problem will be .05.

C. \( F \) is the test statistic.

\[
F = \frac{\text{Estimated variance between the treatments}}{\text{Estimated variance within the treatments}}
\]

Note: Salesperson is the treatment variable and sales is the response variable.

t is the number of treatments
n is the number of rows in a treatment
N is total observations
SS, is the sum of the squares for treatments
SS, is the sum of the squares for error
SS, TOTAL is the total sum of the squares
MS, is the mean squares for treatments
MS, is the mean squares for error

\[
\text{df} = t - 1 = 3 - 1 = 2 \quad \text{for the numerator}
\]

\[
\text{df} = N - t = 12 - 3 = 9 \quad \text{for the denominator}
\]

F’s critical value is 4.26.

D. Reject the null hypothesis when \( F \) from the test statistic is beyond the critical value of \( F \) for the .05 level of significance.

E. Apply the decision rule.

### Variance Analysis Summary Table

<table>
<thead>
<tr>
<th>Variance Sources</th>
<th>df</th>
<th>Sum of the Squares (variance)</th>
<th>ANOVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Treatments</td>
<td>t - 1</td>
<td>( SS_T )</td>
<td>( MS_T = \frac{SS_T}{t-1} )</td>
</tr>
<tr>
<td>Within Treatments (error)</td>
<td>N - t</td>
<td>( SS_E )</td>
<td>( MS_E = \frac{SS_E}{N-t} )</td>
</tr>
<tr>
<td>Total Variance</td>
<td>N - 1</td>
<td>( SS_{TOTAL} )</td>
<td></td>
</tr>
</tbody>
</table>

### Weekly Sales (x) in Thousands of Dollars for 3 Treatments (T)

<table>
<thead>
<tr>
<th>Salesperson</th>
<th>L is ( T_1 )</th>
<th>Salesperson</th>
<th>M is ( T_2 )</th>
<th>Salesperson</th>
<th>N is ( T_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales ( (X_i) )</td>
<td>( X_1^2 )</td>
<td>Sales ( (X_j) )</td>
<td>( X_2^2 )</td>
<td>Sales ( (X_k) )</td>
<td>( X_3^2 )</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>6</td>
<td>36</td>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>8</td>
<td>64</td>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>7</td>
<td>49</td>
<td>6</td>
<td>36</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>Column Totals</td>
<td>24</td>
<td>26</td>
<td>34</td>
<td>( \Sigma X ) = 84</td>
<td></td>
</tr>
<tr>
<td>Required for Calculations</td>
<td>4</td>
<td>16</td>
<td>( \Sigma X ) = 36</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>( (\Sigma X)^2 )</td>
<td>( \Sigma X^2 )</td>
<td>( \Sigma X^2 )</td>
<td>( \Sigma X^2 )</td>
<td>( \Sigma X^2 )</td>
<td></td>
</tr>
<tr>
<td>576</td>
<td>676</td>
<td>1156</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>( N = 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Sigma X^2 )</td>
<td>144</td>
<td>169</td>
<td>289</td>
<td>( \Sigma X^2 ) = 602</td>
<td></td>
</tr>
<tr>
<td>( \Sigma X^2 )</td>
<td>150</td>
<td>172</td>
<td>294</td>
<td>( \Sigma X^2 ) = 616</td>
<td></td>
</tr>
</tbody>
</table>

\[
SS_T = \Sigma \left( \frac{(\Sigma X)^2}{n} \right) - \left( \frac{\Sigma X^2}{N} \right)
\]

\[
= 602 - \frac{84^2}{12}
\]

\[
= 602 - 588 = 14
\]

\[
MS_T = \frac{SS_T}{t-1} = \frac{14}{3-1} = 7.0
\]

\[
SS_E = \Sigma X^2 - \Sigma \left( \frac{(\Sigma X)^2}{n} \right)
\]

\[
= 616 - 602
\]

\[
= 14
\]

\[
MS_E = \frac{SS_E}{N-t} = \frac{14}{12-3} = \frac{14}{9} = 1.56
\]

Total variance equals \( SS_T + SS_E = 14 + 14 = 28 \). Total variance also equals

\[
SS_{TOTAL} = \Sigma X^2 - \frac{(\Sigma X)^2}{N}
\]

\[
= 616 - 588 = 28
\]

Note: Half the variability has been explained by the treatment variable.

Reject \( H_0 \) because \( F = \frac{MS_T}{MS_E} = \frac{7.0}{1.56} = 4.49 \) and 4.49 > 4.26. Mean sales of these salespeople are not equal.