PART V
THE PRESENT AND FUTURE VALUE OF MONEY

Interest is the price paid for the use of money over a period of time. It is logical to think of interest as an accumulation added to an initial value, the principal, with the resulting sum or amount growing larger with time. Compound interest results when interest is paid on interest resulting in much faster growth than with simple interest, where compounding does not take place. With an annuity, growth is even faster as a number of equal payments (principals) are added every equal time period to the compounding process.

PRESENT VALUE is the interest accumulation process in reverse. Rather than adding interest to a principal to determine a sum, it is in effect subtracted from a sum to determine a principal. Your accumulation loses value as you move from some point in the future back towards the present. Value at the beginning of a time line is the Present Value and value at the end of a time line is the Future Value, often called the Sum. These concepts will become more understandable as you study the following practical problems.

INTEREST FORMULAS AND SAMPLE PROBLEMS

Note: Students will find it easier to study the Future Value Analysis on the right before the Present Value Analysis on the left. Problems B. and C. require use of the Tables on the next page.

A. Simple Interest (one payment, one interest calculation) Problem: Calculate the Present Value of $116 to be received in one year and the Future Value in one year of $100 today. Use 16% simple interest.

Given: F = $116  
\[ i = 16\% \]  
\[ n = 1 \text{ year} \]  
\[ P = \text{?} \]

\[ P = F - I = F - (Pin) \]
\[ = $116 - ($100)(.16)(1) \]
\[ = $116 - $16 \]
\[ = $100 \]

Note: $116 future dollars are worth $100 in the present, and $100 of present dollars are $116 future dollars.

B. Compound Interest (one payment, > 1 interest calculation) Problem: Calculate the Present Value of $117 to be received in one year and the Future Value in one year of $100 today. Use 16% interest compounded quarterly.

Given: F = $117  
\[ i = 16\% / 4 = 4\% \]  
\[ n = (1)(4) = 4 \text{ qtrs.} \]  
\[ P = \text{?} \]

\[ P = F(PVM) \]
\[ = 117(.8548) \]
\[ = 100 \]

Given: P = $100  
\[ i = 16\% / 4 = 4\% \]  
\[ n = (1)(4) = 4 \text{ qtrs.} \]  
\[ F = \text{?} \]

\[ F = P(FVM) \]
\[ = 100(1.170) \]
\[ = 117 \]

Note: $117 future dollars are worth $100 in the present, and $100 of present dollars are $117 future dollars.

C. Annuity (> 1 payment, > 1 interest calculation) Problem: Calculate the Present Value and Future Value of four $100 payments, one made every 3 months. Use 16% interest compounded quarterly.

Given A = $100  
\[ i = 16\% / 4 = 4\% \]  
\[ n = (1)(4) = 4 \text{ qtrs.} \]  
\[ P = \text{?} \]

\[ P = A(PVMA) \]
\[ = 100(3.630) \]
\[ = 363.00 \]

Given A = $100  
\[ i = 16\% / 4 = 4\% \]  
\[ n = (1)(4) = 4 \text{ qtrs.} \]  
\[ F = \text{?} \]

\[ F = A(FVMA) \]
\[ = 100(4.246) \]
\[ = 424.60 \]

Note: The $400 in payments are worth less than $400 if brought back and are worth more than $400 if brought forward.
Question: Assume someone won exactly $1,000,000 in their state lottery, 20 payments of $50,000 beginning in one year. Funds invested earned 12% compounded annually. Using the above tables calculate:

1. The value of the annuity today.
2. The value of the annuity if all funds received are invested.
3. What is the value today of your answer to question 2?
4. What is the value in twenty years of your answer to question 3?
5. In actuality your answers are all equal.

Answer: 

GIVEN: \( A = $50,000 \) i = 12% compounded annually, \( n = 20 \) time periods

1. \( P = A(PVMA) \)
   \[ P = ($50,000)(7.469) \]
   \[ P = $373,450 \]

2. \( F = A(FVMA) \)
   \[ F = $50,000(72.052) \]
   \[ F = $3,602,600 \]

3. \( P = F(PVM) \)
   \[ P = $3,602,600(.1037) \]
   \[ P = $373,450 \]

4. \( F = P(FVM) \)
   \[ F = $373,450(9.646) \]
   \[ F = $3,602,600 \]

Question: The interest earned on an investment is called the Internal Rate of Return (IRR). Suppose a $100,000 machine bought today will generate a net return of $20,128.82 per year for 8 years. In this simplified example, you are to assume all expenses and revenues flow at the end of the year and that taxes and depreciation are ignored. To calculate IRR solve \( P = A(PVMA) \) for PVMA.

Answer: 

6. \( P = A(PVMA) \)
   \[ $100,000 = $20,128.82(PVMA) \]
   \[ PVMA = 4.968 \]
   \[ \text{Compounded Annually} \]