Chapter 4 Measuring Dispersion of Ungrouped Data

I. Introduction
A. Dispersion refers to the spread of data, its variability.
B. Dispersion is important because it determines the reliability of central tendency measurements.
C. Comparing the dispersion of different data sets may be revealing. Two students might have the same grade point average with one having all B's and the other having half A's and half C's.
D. This page will explore population parameters. Where sample statistic formulas differ, calculations will be done on the next page.
E. The sample data for self-help rentals presented in chapter 3 will be used here as population data.

II. Range
A. The range is the highest value (H) minus the lowest value (L).
B. \( H - L = 8 - 1 = 7 \)
C. While easy to calculate, the range is severely affected by unusual circumstances. In this case, a snowstorm caused Linda to close early limiting that day's rentals to one unit.

III. Population average deviation (AD)
A. The average deviation is the mean of the absolute values of the deviations from the mean.

\[
AD = \frac{\sum |x - \mu|}{N} = \frac{14}{7} = 2
\]

Note: \( N \) is population size.

B. Using the absolute value of the deviations is necessary because the sum of the deviations is zero.
C. The average deviation is a quick way to measure dispersion. The soon to be explained variance and standard deviation are more valuable measures.

IV. Population variance (\( \sigma^2 \)) and standard deviation (\( \sigma \))
A. The variance solves the problem of the sum of the variations from a mean being zero by squaring the differences.
B. The variance is the average of the squared deviations of the data from their mean.
C. The resulting measure is similar to the averaged deviation although it is larger because the variation was squared.
D. This problem is solved with the standard deviation which is the square root of the variance.
E. The population variance

\[
\sigma^2 = \frac{\sum (x - \mu)^2}{N} = \frac{38}{7} = 5.4
\]

Alternative Formula

\[
\sigma^2 = \frac{\sum x^2}{N} - \left( \frac{\sum x}{N} \right)^2
\]

\[
= \frac{213}{7} - \left( \frac{35}{7} \right)^2
\]

\[
= 30.4 - 8.5 = 5.4
\]

F. Population standard deviation

\[
\sigma = \sqrt{\sigma^2} = \sqrt{5.4} = 2.3
\]